

**Problem Set II: Due Wednesday, October 16, 2013**

Problem Session Discussion: Tuesday, October 15, 2013

*“I do not like to be dunned and teased by foreigners about mathematical things.”*  
- Isaac Newton

- 1a.) Consider a medium with index of refraction  $n = n(x, y)$ . Use Fermat's Principle to derive an equation for the ray along which light propagates in such a medium.
- b.) Take  $n = n(x)$ , such that  $n = n_1$  for  $x < x_0$  and  $n = n_2$  for  $x > x_0$ . Take  $n_2 > n_1$ . Derive Snell's law from Fermat's Principle.
- N.B. Fermat's Principle says light will choose the path of minimal time.

- 2a.) For a particle of mass  $m$  moving thru a potential  $U(x)$ , the time-independent Schrodinger equation is:

$$-\left(\frac{\hbar^2}{2m}\right)\frac{d^2\Psi}{dx^2} + U\Psi = E\Psi.$$

$\Psi(x)$  is the wave function and  $E$  is the energy.  $\Psi^*$  is the complex conjugate to  $\Psi$ .

Show that the Schrodinger equation is the Lagrange Equation for:

$$L = \frac{-\hbar^2}{2m} \left| \frac{d\Psi}{dx} \right|^2 - \Psi^*(U - E)\Psi.$$

- b.) In this spirit, how would you go about showing conservation of probability?
- 3a.) In the case  $L = L(q, \dot{q}, \ddot{q}, t)$ , derive the corresponding equation of motion.
- b.) How does one formulate energy conservation for such a system? Begin from the requirement of homogeneity in time.

- 4.) What is the shape of the hill down which you could ski without friction such that you would descend in the least time? Assume energy is conserved and that the only force acting is that of gravity.
- 5.) Consider a system with Lagrangian in Cartesian coordinates which is given by:

$$L = \frac{1}{2} \sum_{\alpha} m_{\alpha} (\dot{x}_{\alpha}^2 + \dot{y}_{\alpha}^2 + \dot{z}_{\alpha}^2) - U.$$

Now, if this is more conveniently expressed in terms of generalized coordinates  $(q_1, q_2 \dots q_s)$ , use the transformations:

$$x_a = f_a(q_1, q_2 \dots q_s)$$

$$\dot{x}_a = \sum_k \frac{\partial f_a}{\partial q_k} \dot{q}_k$$

to rewrite the Lagrangian in terms of generalized coordinates.

- a.) What is the most general form of  $L$  in terms of the  $q$ 's?
- b.) Specify the form of the kinetic energy as fully as possible.
- c.) Derive the Lagrangian EOMs.